Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

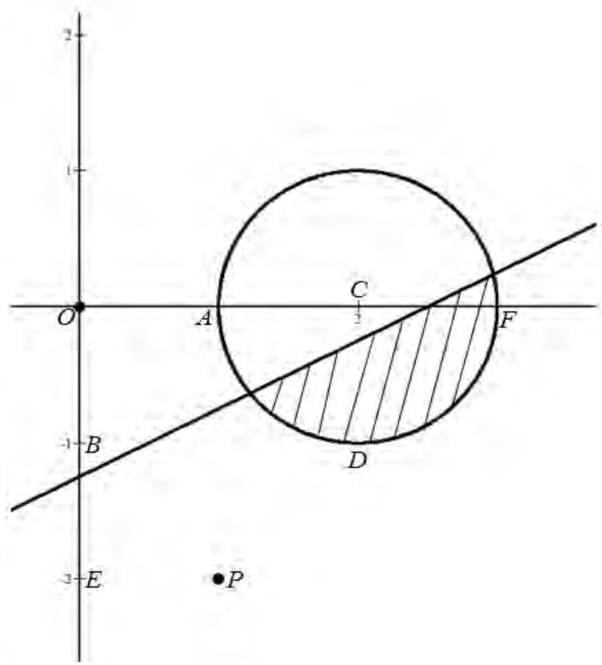
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $-0.5 < 2^{x+1} - 2 < 0.5$, can be in two separate statements, or $\left(2^{x+1}-2\right)^2 < 0.5^2$ or corresponding pair of linear equations $0.5 = 2^{x+1} - 2$ and $-0.5 = 2^{x+1} - 2$ or quadratic equation $\left(2^{x+1}-2\right)^2 = 0.5^2$	B1	$-0.25 < 2^x - 1 < 0.25$, can be in two separate statements, or $\left(2^x-1\right)^2 < 0.25^2$ or corresponding pair of linear equations $0.25 = 2^x - 1$ and $-0.25 = 2^x - 1$ or quadratic equation $\left(2^x-1\right)^2 = 0.25^2$. Incorrect inequality mark recoverable by correct final answer or $x < 0.32$ and $x > -0.42$.
	Use correct method for solving an equation or inequality of the form $2^{x+1} = a$ or $2^x = b$ where $a, b > 0$	M1	Reach $(x + 1)\ln 2 = \ln a$ or equivalent, do not need to reach $x = \ldots$
	Obtain critical values $x = 0.322$ and -0.415 or awrt $x = 0.32$ and -0.42 or exact equivalents	A1	e.g. $\frac{\ln 2.5}{\ln 2} - 1$ and $\frac{\ln 1.5}{\ln 2} - 1$.
	State final answer $-0.415 < x < 0.322$ or $(-0.415, 0.322)$	A1	Need 3 significant figures. Need combined result, not $x < 0.32$ and $x > -0.42$. Must be strict inequalities. No working, 0/4.
	Alternative method for Question 1		
	Use correct method for solving an equation or inequality of the form $2^{x+1} = a$ or $2^x = b$ where $a, b > 0$	M1	May see $2^{x+1} = 1.5$ and $2^{x+1} = 2.5$. Reach $(x + 1)\ln 2 = \ln a$ or equivalent, don't need to reach $x = \ldots$
	Obtain one critical value, e.g. 0.322 or awrt $x = 0.32$ or exact equivalent	A1	e.g. $\frac{\ln 2.5}{\ln 2} - 1$.
	Obtain the other critical value e.g. -0.415 or awrt $x = -0.42$ or exact equivalent	A1	e.g. $\frac{\ln 1.5}{\ln 2} - 1$.

Question	Answer	Marks	Guidance
1	State final answer $-0.415 < x < 0.322$ or $(-0.415, 0.322)$	**A1**	Need 3 significant figures. Need combined result, not $x < 0.32$ and $x > -0.42$. Must be strict inequalities. No working, 0/4.
		4	

Question	Answer	Marks	Guidance
2	Show a circle centre (2, 0)	**B1**	
	Show the relevant part of a circle with radius 1	**B1 FT**	FT centre not at the origin even if centre at 1 – 2i. Must clearly go through (1, 0) or (3, 0) (oe for FT mark).
	Show the point representing 1 – 2i	**B1**	Can be implied by correct perpendicular bisector
	Show the perpendicular bisector of the line joining 1 – 2i and the origin. Perpendicular to *OP* by eye and at midpoint of *OP* by eye sufficient. Must reach midpoint of *OP* and if extended will cut *BE*.	**B1 FT**	FT on the position of 1 – 2i.

Question	Answer	Marks	Guidance
2	Shade the correct region. Dependent on all previous marks, except in case 3 below, and the perpendicular must cut axes between *CF* and *BE,* but not actually through *C* or *F* and not through *B* or *E* Scale can be implied by dashes 1 Scale only on y-axis and $2OA = OC$ B1, B1FT, B1, B1FT, B1 2 Scale only on x-axis and $2OB = OE$ B1, B1FT, B1, B1FT, B1 3 No scale on either axis, but $2OA = OC$ B0, B1FT, B0, B1FT, B1 then $2OB = OE$	**B1**	
		5	

Question	Answer	Marks	Guidance
3	$2(-2)^3 + a(-2)^2 + b(-2) + 6 = -38$ Allow errors $x+2 \overline{)\,2x^3 + ax^2 \quad + bx \quad + 6}$ with quotient $2x^2 + (a-4)x + b - 2a + 8$ $\underline{2x^3 + 4x^2}$ $(a-4)x^2 + bx$ $\underline{(a-4)x^2 + (2a-8)x}$ $(b-2a+8)x + 6$ $\underline{(b-2a+8)x + 2b - 4a + 16}$ $4a - 2b - 10$	**M1**	Substitute $x = -2$ and equate the result to -38 or divide by $x + 2$ to obtain quadratic quotient, and equate constant remainder to -38.

Question	Answer	Marks	Guidance
3	Obtain a correct evaluated equation, e.g. $-16 + 4a - 2b + 6 = -38$ or $4a - 2b = -28$	**A1**	
	$2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 6 = \frac{19}{2}$ Allow errors $2x-1 \,\overline{)\, 2x^3 + ax^2 + bx + 6}$ with quotient $x^2 + \frac{a+1}{2}x + \frac{b}{2} + \frac{a}{4} + \frac{1}{4}$ $2x^3 - x^2$ $(a+1)x^2 + bx$ $(a+1)x2 - \left(\frac{a}{2} + \frac{1}{2}\right)x$ $\left(b + \frac{a}{2} + \frac{1}{2}\right)x + 6$ $\left(b + \frac{a}{2} + \frac{1}{2}\right)x - \left(\frac{b}{2} + \frac{a}{4} + \frac{1}{4}\right)$ $6 + \frac{b}{2} + \frac{a}{4} + \frac{1}{4}$	**M1**	Substitute $x = \frac{1}{2}$ and equate the result to $\frac{19}{2}$ or divide by $2x - 1$ to obtain quadratic quotient, and equate constant remainder to $\frac{19}{2}$.
	Obtain a correct evaluated equation, e.g. $\frac{1}{4} + \frac{a}{4} + \frac{b}{2} + 6 = \frac{19}{2}$ or $\frac{a}{4} + \frac{b}{2} = \frac{13}{4}$	**A1**	
	Obtain $a = -3$ and $b = 8$	**A1**	ISW
		5	

Question	Answer	Marks	Guidance
4	$\dfrac{2 \pm \sqrt{(-2)^2 - 4(3+i)(3-i)}}{2(3+i)}$	M1	Use quadratic formula to solve for w
	Use $i^2 = -1$ in $(3+i)(3-i)$	M1	
	Obtain one of the answers $w = \dfrac{2+6i}{6+2i}$ or $w = \dfrac{2-6i}{6+2i}$	A1	Must be simplified to this form.
	Show **intention** to multiply numerator and denominator by conjugate of their denominator.	M1	Independent of previous M marks but must be of the same form, e.g. $\dfrac{a}{b+ci}$.
	Obtain final answers $\dfrac{3}{5}+\dfrac{4}{5}i$ and $-i$ Accept $0.6 + 0.8i$ and $0 - i$	A1	**SC** Both correct final answers from $w = \dfrac{2+6i}{6+2i}$ and $w = \dfrac{2-6i}{6+2i}$ seen, no evidence of conjugate, then **SC B1** for both. Allow $x = \dfrac{3}{5}$, $y = \dfrac{4}{5}$ or $x = 0$, $y = -1$. A0 for $\dfrac{3+4i}{5}$.
	Alternative method for Question 4		
	Multiply the equation by $3 - i$	M1	
	Use $i^2 = -1$ in $(3+i)(3-i)$	M1	
	Obtain $10w^2 - 2(3-i)w + (3-i)^2 = 0$ or equivalent	A1	
	Use quadratic formula or factorise to solve for w	M1	

Question	Answer	Marks	Guidance
4	Obtain final answers $\frac{3}{5}+\frac{4}{5}\mathrm{i}$ and $-\mathrm{i}$ Accept $0.6 + 0.8\mathrm{i}$ and $0 - \mathrm{i}$	A1	**SC** Both correct final answers from $10w^2 - 2(3 - \mathrm{i})w + (3 - \mathrm{i})^2 = 0$ with no working then **SC B1** for both. Allow $x = \frac{3}{5}$, $y = \frac{4}{5}$ or $x = 0$, $y = -1$. A0 for $\frac{3+4\mathrm{i}}{5}$.
	Alternative method for Question 4		
	Substitute $w = x + \mathrm{i}y$ and form equations for real and imaginary parts	M1	
	Use $\mathrm{i}^2 = -1$ in $(x + \mathrm{i}y)^2$	M1	
	Obtain $3(x^2 - y^2) - 2xy - 2x + 3 = 0$ and $x^2 - y^2 + 6xy - 2y - 1 = 0$	A1	OE
	Form quartic equation in x only or y only using the correct substitution and solve for x or y	M1	Use correct $y = \frac{(3-x)}{10x-3}$ to attempt to form and solve $50x^4 - 60x^3 + 63x^2 - 27x = 0$ $x(5x - 3)(10x^2 - 6x + 9) = 0$. Use correct $x = \frac{3(1+y)}{10y+1}$ to attempt to form and solve $100y^4 + 40y^3 - 66y^2 - 14y - 8 = 0$ $(y + 1)(5y - 4)(20y^2 + 4y + 2) = 0$.
	Obtain final answers $\frac{3}{5}+\frac{4}{5}\mathrm{i}$ and $-\mathrm{i}$ Accept $0.6 + 0.8\mathrm{i}$ and $0 - \mathrm{i}$	A1	**SC** Both correct final answers from $3(x^2 - y^2) - 2xy - 2x + 3 = 0$ and $x^2 - y^2 + 6xy - 2y - 1 = 0$ with no working then **SC B1** for both. Allow $x = \frac{3}{5}$, $y = \frac{4}{5}$ or $x = 0$, $y = -1$. A0 for $\frac{3+4\mathrm{i}}{5}$.
		5	

Question	Answer	Marks	Guidance
5	Use correct product or quotient rule	**M1**	Need attempt at both derivatives condone errors in chain rule. In quotient rule allow BOD in formula if $\pm 2x$ seen unless clear that incorrect formula has been used. If omit denominator or forget to square or complete reversal of signs then M0 A0 M1 A1 A1 A1.
	Obtain correct derivative in any form, e.g. $\dfrac{6x(1-x^2)e^{3x^2-1}+2xe^{3x^2-1}}{\left(1-x^2\right)^2}$	**A1**	If $6x\left(1-x^2\right)e^{3x^2-1}+2xe^{3x^2-1}=0$ from the start, with no wrong formula seen, award M1A1.
	Equate derivative (or its numerator) to zero and solve for x	**M1**	$6x - 6x^3 + 2x = 0$ and solve. Allow for just one x value. Allow if from solution of 3 term quadratic equation, but if they get $x = 0$ the x must factorise out
	Obtain the point $(0, e^{-1})$ or exact equivalent	**A1**	Or for all three x coordinates found 0, $\pm\dfrac{2\sqrt{3}}{3}$ oe and no extras but if this is the case then one pair of correct coordinates A1 and both other pairs of correct coordinates A1. Accept, e.g. $x = 0, y = e^{-1}$ ISW for last 3 marks.
	Obtain the point $\left(\dfrac{2\sqrt{3}}{3}, -3e^3\right)$ or exact equivalent	**A1**	Allow √(4/3).
	Obtain the point $\left(-\dfrac{2\sqrt{3}}{3}, -3e^3\right)$ or exact equivalent	**A1**	
		6	

Question	Answer	Marks	Guidance
6(a)	Use correct Pythagoras $\cot^2\theta = \operatorname{cosec}^2\theta - 1$ or $\cot^2\theta = 1/\sin^2\theta - 1$ or $\cot^2\theta = \cos^2\theta/\sin^2\theta$ and then $\cos^2\theta = 1 - \sin^2\theta$, together with double angle formula $\cos 2\theta = 1 - 2\sin^2\theta$, to obtain an equation in $\sin\theta$ or $\sin\theta$ and $\operatorname{cosec}^2\theta$	**M1**	If consistent omission of brackets, e.g. $(\sin\theta)^2$ written as $\sin\theta^2$ then **SC B1** in place of M1A1.
	Obtain a correct equation in $\sin\theta$ in any form	**A1**	e.g. $1/\sin^2\theta - 1 + 2(1 - 2\sin^2\theta) = 4$ or $\dfrac{1-\sin^2}{\sin^2} + 2\left(1-2\sin^2\right) = 4$. If $\dfrac{\cos^2}{\sin^2} + 2\left(1-2\sin^2\right) = 4$ then e.g. $1-\sin^2 + 2\left(1-2\sin^2\right)\sin^2 = 4$. (missing $\sin^2$ on right) allow M1A1A0.
	Reduce to the given answer of $4\sin^4\theta + 3\sin^2\theta - 1 = 0$ correctly	**A1**	AG Must follow from a horizontal equation (no denominators). If $s = \sin\theta$ used and defined, allow all marks. If not defined, award M1A1A0.
		3	

Question	Answer	Marks	Guidance
6(b)	Solve the given quadratic to obtain a value for θ	M1	$(4\sin^2\theta - 1)(\sin^2\theta + 1) = 0$ and solve for θ. Incorrect sign in solution of quadratic seen, e.g. $(4\sin^2\theta - 1)(\sin^2\theta - 1) = 0$ then M0 A0 A0 but if only see $(4\sin^2\theta - 1) = 0$ and nothing incorrect seen allow 3/3.
	Obtain answer, e.g. $\theta = 30°$	A1	$\pi/6$ award A0
	Obtain three further answers, e.g. $\theta = 150°$, $210°$ and $330°$ and no others in the interval	A1	Ignore any answers outside interval. $5\pi/6$ $7\pi/6$ $11\pi/6$ award A1.
		3	

Question	Answer	Marks	Guidance
7(a)	State or imply $2y\frac{dy}{dx}$ as the derivative of y^2	B1	Allow for $3x^2dx + 2ydy$ or $F_x = 3x^2 + 6x$ and $F_y = 2y + 3$.
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	$3x^2 + 2y\frac{dy}{dx} + 6x + 3\frac{dy}{dx} = 0$ or $3x^2dx + 2ydy + 6xdx + 3dy = 0$ or $\frac{dy}{dx} = -\frac{F_x}{F_y}$ need evidence from B1 mark or formula must be seen. Allow errors.
	Obtain the given answer	A1	AG $\frac{dy}{dx} = -\frac{3x^2 + 6x}{2y+3}$ not $\frac{-3x^2 - 6x}{2y+3}$. Must factorise with $\frac{dy}{dx}$ e.g. $3x^2 + 6x + \frac{dy}{dx}(2y+3) = 0$ or $3x^2dx + 6xdx + dy(2y+3) = 0$.
		3	

Question	Answer	Marks	Guidance
7(b)	Equate numerator to zero and solve for x	*M1	Allow for just one x value.
	Obtain $x = 0$ and $x = -2$ only	A1	
	Substitute their x, [$x = 0$ or $x = -2$] in curve equation to obtain quadratic equation in y equal to 0	DM1	$y^2 + 3y - 4 = 0$ or $y^2 + 3y = 0$.
	Obtain $y = 1$ and $y = -4$ [when $x = 0$]	A1	
	Obtain $y = 0$ and $y = -3$ [when $x = -2$]	A1	ISW If forget $x = 0$ then max 3/5.
		5	

Question	Answer	Marks	Guidance
8	Separate variables correctly and reach $a\sec^2 3y$ or be^{-4x}	**B1**	Condone missing integral signs or dy and dx, but allow if recognisable integrals follow. Not for $1/\cos^2 3y$ and $1/e^{4x}$.
	Obtain term $-\frac{1}{4}e^{-4x}$	**B1**	Can recover the previous B1 if de^{-4x} seen here.
	Obtain only a term of the form $a\tan 3y$	**M1**	Can recover the first B1 if $a\tan 3y$ seen here.
	Obtain term $\frac{1}{3}\tan 3y$	**A1**	
	Use $x = 2$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a\tan by$ and $ce^{\pm 4x}$	**M1**	May see $\tan by$ and $e^{\pm 4x}$ here.
	Obtain correct answer in any form	**A1**	e.g. $\frac{1}{3}\tan 3y = -\frac{1}{4}e^{-4x} + \frac{1}{4}e^{-8}$ or $\frac{1}{3}\tan 3y = -\frac{1}{4}e^{-4x} + 8.39 \times 10^{-5}$
	Obtain final answer $y = \frac{1}{3}\tan^{-1}\left(\frac{3}{4}e^{-8} - \frac{3}{4}e^{-4x}\right)$	**A1**	ISW OE e.g. $y = \frac{1}{3}\tan^{-1}\left(2.52\times10^{-4} - \frac{3}{4}e^{-4x}\right)$
		7	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{Ax+B}{2+3x^2}+\dfrac{C}{2-x}$	**B1**	If incorrect partial fractions e.g. $A = 0$ or $Ax^2 + B$ then M1, A1 A0 for correct C. Only allow single A1 even if other coefficients correct. B1 recoverable by a correct form end statement.
	Use a correct method for finding a coefficient	**M1**	e.g. $(Ax + B)(2 - x) + C(2 + 3x^2)$ $= (3C - A)x^2 + (2A - B)x + (2B + 2C)$ $= 17x^2 - 7x + 16$.
	Obtain one of $A = -2$, $B = 3$ and $C = 5$	**A1**	If error present in above still allow A1 for C.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Extra term in partial fractions, $D/(2 + 3x^2)$, that is 4 unknowns A, B, C and D then B0 unless recover at end, e.g. by setting B or $D = 0$. If B or D set to any value other than 0 and all coefficients correctly found to their new values then allow all A marks, but still B0 for partial fraction expression unless $B + D$ combined. Hence A1 for each coefficient, but nothing for coefficient set to specific value. Another case of extra term in partial fraction expression, namely $+ F$, mark as above but need $F = 0$ to recover B1.
		5	

Question	Answer	Marks	Guidance
9(b)	Use a correct method to find the first two terms of the expansion $(2-x)^{-1} = 2^{-1} + (-1)\,2^{-2}(-x) + [(-1)(-2)2^{-3}(-x)^2/2!]$, $\left(1+\frac{3x^2}{2}\right)^{-1} = 1 - \frac{3x^2}{2}$ or $\left(1-\frac{x}{2}\right)^{-1} = 1 - \left(\frac{-x}{2}\right)$	**M1**	Symbolic coefficients are not sufficient for the M1.
	$\frac{Ax+B}{2}\left[1+(-1)\frac{3x^2}{2}\ldots\right]$ $A=-2 \quad B=3$ $\frac{C}{2}\left[1+(-1)\left(\frac{-x}{2}\right)+\frac{(-1)(-2)}{2}\left(\frac{-x}{2}\right)^2 + \frac{(-1)(-2)(-3)}{6}\left(\frac{-x}{2}\right)^3\ldots\right]$ $C=5$	**A1 FT**	Obtain correct un-simplified expansions up to the term in x^3 of each partial fraction.
	$=\frac{3-2x}{2}\left(1-\frac{3x^2}{2}\right)+\frac{5}{2}\left(1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}\right)$ $=\left(\frac{3}{2}+\frac{5}{2}\right)+\left(-1+\frac{5}{4}\right)x+\left(-\frac{9}{4}+\frac{5}{8}\right)x^2+\left(\frac{3}{2}+\frac{5}{16}\right)x^3$	**A1 FT**	Unsimplified $(2-x)^{-1}$ expanded correctly, error in simplifying before their C is involved in the expression, allow A1FT when their C is introduced. The FT is on A, B, C.
	Multiply expansion of $\left(1+\frac{3x^2}{2}\right)^{-1}$ $\left(\text{must reach } 1\pm\frac{3x^2}{2}\right)$ by $Ax+B$, where $AB \neq 0$, up to the term in x^3. Allow if used $Cx+D$ ($Ax+B$ miscopied).	**M1**	Allow either ± 2 or $\pm 2^{-1}$ outside bracket or missing. Allow one error in actual multiplication to acquire the 4 terms [all terms needed]. Ignore errors in higher powers.

Question	Answer	Marks	Guidance
9(b)	Obtain final answer $4+\frac{1}{4}x-\frac{13}{8}x^2+\frac{29}{16}x^3$, or equivalent [If final answer has been multiplied throughout, e.g. by 16 then A0 at the end]	**A1**	Maclaurin's Series: f(0) = 4 B1 f'(0) = 1/4 B1. f''(0) = –13/4 and f'''(0) = 87/8 B1. $4+\frac{1}{4}x-\frac{\frac{13}{4}x^2}{2}+\frac{\frac{87}{8}x^3}{6}$ or equivalent M1 A1. If $1+\frac{1}{4}x-\frac{\frac{13}{4}x^2}{2}+\frac{\frac{87}{8}x^3}{6}$ then M0 A0 unless their f(0) actually is 1.
		5	
9(c)	State answer $\lvert x\rvert<\sqrt{\frac{2}{3}}$ or $-\sqrt{\frac{2}{3}}<x<\sqrt{\frac{2}{3}}$ clear conclusion required	**B1**	Or exact equivalent. Strict inequality.
		1	

Question	Answer	Marks	Guidance
10(a)	Use the product rule correctly on $y = x\cos 2x$	**M1**	dx/dx cos 2x + x d/dx(cos 2x) attempted.
	Obtain the correct derivative in any form	**A1**	e.g. cos 2x – 2x sin 2x. If cos 2x + x–2sin 2x, not recovered, max M1A0A1FTA0 but can recover for full marks by seeing correct substitution.
	Obtain $y=-\frac{\pi}{2}$ and $\frac{dy}{dx}=-1$ when $x=\frac{\pi}{2}$	**A1FT**	FT *their* $\frac{dy}{dx}$ with $x=\frac{\pi}{2}$ substituted.
	Obtain answer $x + y = 0$	**A1**	OE CWO Need to see y and dy/dx at $x=\frac{\pi}{2}$.
		4	

Question	Answer	Marks	Guidance
10(b)	Integrate by parts and reach $ax\sin 2x + b\int \sin 2x\,dx$	*M1	
	Obtain $\frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x\,dx$	A1	OE
	Complete integration and obtain $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$	A1	OE
	Use limits of $x = 0$ and $x = \frac{\pi}{4}$ in the correct order, having integrated twice to obtain $ax \sin 2x + c\cos 2x$	DM1	If correct, $\frac{1}{2}\left(\frac{\pi}{4}\right)\sin\frac{2\pi}{4} + \frac{1}{4}\cos\frac{2\pi}{4} - \frac{1}{4}\cos 0$ or $\frac{1}{2}\left(\frac{\pi}{4}\right)\sin\frac{2\pi}{4} - \frac{1}{4}\cos 0$. Max one substitution error.
	Obtain answer $\frac{\pi}{8} - \frac{1}{4}$ or exact simplified two term equivalent	A1	ISW Accept $\frac{\pi - 2}{8}$. Accept $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$ then final answer.
		5	

Question	Answer	Marks	Guidance
11(a)	Use correct process for modulus on direction vector of l, e.g. $\sqrt{(-1)^2 + 1^2 + 2^2}$	M1	SOI Allow -1^2. Allow $\sqrt{(-\lambda)^2 + \lambda^2 + (2\lambda)^2}$.
	$[\pm]\frac{1}{\sqrt{6}}(-\mathrm{i} + \mathrm{j} + 2\mathrm{k})$	A1	OE Allow coordinates as row or column, but not row or column with **i**, **j** and **k** included.
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>11(b)</td><td>Use a correct method to form an equation for line m</td><td>M1</td><td>Allow even if all signs of point incorrect, namely use $+2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ or $-3\mathbf{i} + \mathbf{j} - \mathbf{k}$.</td></tr>
<tr><td></td><td>Obtain $\mathbf{r} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \mu_1(-5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$</td><td>A1</td><td>OE, e.g. $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} + \mu_2(-5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$
Must have $\mathbf{r} = \ldots$</td></tr>
<tr><td></td><td></td><td>2</td><td></td></tr>
<tr><td>11(c)</td><td>Justify lines are not parallel</td><td>B1</td><td>$(-5, 3, -2) \neq d(-1, 1, 2)$ or $(-5, 3, -2)\mathbf{x}(-1, 1, 2) \neq 0$.
Can find angle (105°, 74.6°, 1.84^c or $1.3(0)^c$) instead but if incorrect B0 and A0 at end.
Accept direction vectors don't have common factor
but not direction vectors are not equal or direction vectors are different or $\mu \neq \lambda$ or scalar product $\neq 0$.
Not the line equations are not multiples of each other.</td></tr>
<tr><td></td><td>Express l or m in component form
e.g. $(-2 - 5\mu_1, 2 + 3\mu_1, -1 - 2\mu_1)$ or $(3 - 5\mu_2, -1 + 3\mu_2, 1 - 2\mu_2)$ or
$(1 - \lambda, -2 + \lambda, -3 + 2\lambda)$</td><td>B1</td><td></td></tr>
<tr><td></td><td>Equate two pairs of components of general points on l and their m and solve simultaneously for λ or for μ</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain correct answer for λ or μ, e.g. $\lambda = \frac{11}{2}$, $\mu_1 = \frac{1}{2}$</td><td>A1</td><td></td></tr>
<tr><td></td><td>Determine that all three equations are not satisfied and the lines fail to intersect and conclude the lines are skew.
Conclusion needs to follow correct working</td><td>A1</td><td><table>
<tr><th>1</th><th>λ</th><th>μ_1</th><th></th><th>2</th><th>λ</th><th>μ_2</th><th></th></tr>
<tr><td>ij</td><td>11/2</td><td>1/2</td><td>$8 \neq -2$</td><td>ij</td><td>11/2</td><td>3/2</td><td>$8 \neq -2$</td></tr>
<tr><td>ik</td><td>4/3</td><td>–1/3</td><td>$-2/3 \neq 1$</td><td>ik</td><td>4/3</td><td>2/3</td><td>$-2/3 \neq 1$</td></tr>
<tr><td>jk</td><td>7/4</td><td>–3/4</td><td>$-3/4 \neq 7/4$</td><td>jk</td><td>7/4</td><td>1/4</td><td>$-3/4 \neq 7/4$</td></tr>
</table>
Dependent on 4 previous marks gained.</td></tr>
<tr><td></td><td></td><td>5</td><td></td></tr>
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